Write your name here


## Mathematics AO3 <br> Mathematical problem solving

Grades 6-9

## Time: 60 minutes

Paper Reference
1MA1

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided
 - there may be more space than you need.
- Calculators must not be used in questions marked with an asterisk (*).
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must show all your working out with your answer clearly identified at the end of your solution.


## Information

- This gold test is aimed at students targeting grades 6-9.
- This test has 9 questions. The total mark for this paper is 48.
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

1. Pavel has a combination lock.

Pavel has to set each part of the lock to a digit between 0 and 9 inclusive. One possible way to do this is shown in the diagram.

(a) How many different ways can Pavel do this?

Pavel decides that the 1st and 3rd digits will be odd numbers and that the 2nd and 4th digits will be even numbers greater than 0 .
(b) How many different ways are possible now?
2. $\mathbf{C}$ is the curve with equation

$$
y=x^{2}-4 x+4
$$

$\mathbf{L}$ is the straight line with equation

$$
y=2 x-4
$$

$\mathbf{L}$ intersects $\mathbf{C}$ at two points, $A$ and $B$.
Calculate the exact length of $A B$.
3. Here is a solid bar made of metal.

The bar is in the shape of a cuboid.
The height of the bar is $h \mathrm{~cm}$.
The base of the bar is a square of side $d \mathrm{~cm}$.
The mass of the bar is $M \mathrm{~kg}$.
$d=8.3$ correct to 1 decimal place.
$M=13.91$ correct to 2 decimal places.
$h=84$ correct to the nearest whole number.


Find the value of the density of the metal to an appropriate degree of accuracy. Give your answer in $\mathrm{g} / \mathrm{cm}^{3}$.

You must explain why your answer is to an appropriate degree of accuracy.
4. (a) Expand and simplify
$x(x+1)(x-1)$

In a list of three consecutive positive integers at least one of the numbers is even and one of the numbers is a multiple of 3 .
$n$ is a positive integer greater than 1.
(b) Prove that $n^{3}-n$ is a multiple of 6 for all possible values of $n$.
$2^{61}-1$ is a prime number.
(c) Explain why $2^{61}+1$ is a multiple of 3 .
*5. The diagram shows triangle $A B C$.


The area of triangle $A B C$ is $k \sqrt{3} \mathrm{~cm}^{2}$.
Find the exact value of $k$.
6. Alfred studies animal populations on an island.

The size of an animal population at the start of 2014 was 2500 .
The size of this animal population increases exponentially.
Alfred assumes that the rate of increase is 20\% per year.
(a) Using his assumption, work out the size of this animal population at the start of 2009.
(b) Alfred's assumption is too high. Explain how your answer to part (a) is affected.
7. A rectangular sheet of paper can be cut into two identical rectangular pieces in two different ways.


When the original sheet of paper is cut one way, the perimeter of each of the two pieces is 50 cm .

When the original sheet of paper is cut the other way, the perimeter of each of the two pieces is 64 cm .

What is the perimeter of the original sheet of paper?
8. The quantity of heat, $H$ calories, delivered by an electric current, $I$ amps, acting for $t$ seconds to heat an amount of water is given by the formula

$$
H=a t I^{2}-b
$$

where $a$ and $b$ are constants.
(a) Rearrange the formula to make $I$ the subject.

The graph gives information about the variation in the temperature, in ${ }^{\circ} \mathrm{C}$, of an amount of water that is allowed to cool from $80^{\circ} \mathrm{C}$.

(b) (i) Work out the average rate of decrease of the temperature of the water between $t=0$ and $t=800$.

The instantaneous rate of decrease of the temperature of the water at time $T$ seconds is equal to the average rate of decrease of the temperature of the water between $t=0$ and $t=800$.
(ii) Find an estimate for the value of $T$.

You must show how you got your answer.
9.


Here is a spinner.
When the arrow is spun once, a 1 or a 2 or a 3 can be scored.
Bill is going to spin the arrow twice.
He will work out his total score by adding the two scores he gets on the two spins.
The probability that he will get a total score of 4 is $\frac{16}{81}$
Assuming that the thickness of the three lines between the sectors may be ignored,
Work out the value of $x$.

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| Question | Working | Answer | Mark | AO | Notes |
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| 1 (a) | $10 \times 10 \times 10 \times 10$ | 10000 | M | 1.3a | M1 $10 \times 10 \times 10 \times 10$ |
|  |  |  | A | 1.3a | A1 cao |
| (b) | $5 \times 4 \times 5 \times 4$ | 400 | M | 1.3a | M1 $5 \times 4 \times 5 \times 4$ |
|  |  |  | A | 1.3a | A1 cao |
| 2 | $\begin{aligned} & 2 x-4=x^{2}-4 x+4 \\ & x^{2}-6 x+8=0 \end{aligned}$ | $\sqrt{20}$ | P | 3.1b | P1 for a process to eliminate $y$, e.g. $2 x-4=x^{2}-4 x+4$ followed by reduction to 3 term quadratic |
|  | $(x-4)(x-2)=0$ |  | P | 3.1b | P1 for factorisation or formula for a 3 term quadratic $=0$ |
|  | $x=4, \quad x=2$ |  | P | 3.1b | P1 for a process to find the values of y |
|  | When $x=4, y=4$ |  | A | 1.3b | A1 all 4 values ( $x=4, y=4$, and $x=2, y=0$ ) |
|  | $\begin{aligned} & \text { When } x=2, y=0 \\ & 4-2=2 \end{aligned}$ |  | P | 3.1a | P1 for a correct process to find the distance ${ }^{2}$ or distance between the 2 points, e.g. ( 4 ' - '2') $2+\left({ }^{\prime} 4^{\prime}-{ }^{\prime} \mathbf{0}^{\prime}\right) 2$ |
|  | $\begin{aligned} & 4-0=4 \\ & 22+42 \end{aligned}$ |  | A | 1.3a | A1 $\sqrt{ } 20$ |


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| 3 | $\frac{1000 \times 13.915}{8.25^{2} \times 83.5}=2.448$ | $2.4 \mathrm{~g} / \mathrm{cm}^{3}$ | B | 1.1 | B1 for $83.5 \leq h<84.5$ or $8.25 \leq d<8.35$ <br> (or correct bounds) or $13.905 \leq M<13.915$ <br> (or correct bounds). <br> Accept $h=84.5$ or $d=8.35$ or $M=13.915$ |
|  | $\frac{1000 \times 13.905}{8.35^{2} \times 84.5}=2.360$ |  | P | 3.1c | P1 for correct process to find upper bound of $\mathrm{D}(=2.4(48 \ldots$ or $0.0024(48 \ldots)$ ) oe |
|  |  |  | P | 3.1c | P1 for correct process to find lower bound of $D(=2.3$ ( $60 \ldots$ or $0.0023(6 \ldots))$ oe |
|  |  |  | P | 2.4a | P1 for an explanation or a correct process to find D to an appropriate degree of accuracy |
|  |  |  | A | 1.3a | A1 $2.4 \mathrm{~g} / \mathrm{cm}^{3}$ |
| 4 (a) | $2^{61}-1$ is prime so not a multiple of 3 $2^{61}$ is not a multiple of 3 Hence $2^{61}+1$ must be a multiple of 3 | Shown | M | 1.3a | M1 for $x\left(x^{2}-1\right)$ or $\left(x^{2}+{ }^{x}\right)\left({ }^{x}-1\right)$ oe |
|  |  |  | A | 1.3a | A1 cao |
| (b) |  | Shown | P | 2.4 b | P1 for explanation to show that $n^{3}-n$ is the product of three consecutive positive integers, e.g. $n^{3}-n=(n-1) n(n+1)$ |
|  |  |  | C | 2.4 b | C1 for a correct conclusion to the proof, e.g. at least one of these is even and one is a multiple of 3 so the product is a multiple of 6 |
| (c) |  | Shown | P | 2.4a | P1 for recognising that $2^{61}-1,2^{61}$ and $2^{61}+1$ are three consecutive positive integers |
|  |  |  | C | 2.4a | C 1 for a convincing argument |


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| *5 | $\begin{aligned} & (2 x-1)^{2}= \\ & (x+1)^{2}+(x-1)^{2}-2(x+1)(x-1) \times \\ & \cos 120^{\circ} \end{aligned}$ | $\frac{15}{4}$ | B | 1.1 | B1 for correct statement of cosine rule or $\frac{1}{2} a b \sin C$ |
|  | $\begin{aligned} & 4 x^{2}-4 x+1=x^{2}+2 x+1+x^{2}-2 x+ \\ & 1-2\left(x^{2}-1\right) \times(-0.5) \\ & x^{2}-4 x=0 \end{aligned}$ |  | P | 3.2 | P1 for strategy to start to solve problem, e.g. $(2 x-1)^{2}=(x+1)^{2}+(x-1)^{2}-2(x+1)(x-1) \times$ $\cos 120^{\circ}$ |
|  | Area $=0.5 \times 3 \times 5 \times \sin 120^{\circ}$ |  | P | 3.2 | P1 for strategy to reduce to a quadratic equation, e.g. $x^{2}-4 x=0$ |
|  |  |  | M | 1.3b | M1 for method to solve quadratic equation |
|  |  |  | P | 3.2 | P1 for attempt to use $0.5 a b \sin C$ with numeric or algebraic values substituted |
|  |  |  | P | 3.2 | P1 for process to equate to $k \sqrt{ } 3$ |
|  |  |  | A | 1.3b | A1 for $k=\frac{15}{4}$ oe |
| 6 (a) | $2500=P \times 1.20^{5}$ | 1005 | P | 3.1c | P1 for process to translate problem into algebraic form, e.g. $2500=P \times 1.20^{5}$ |
|  | $P=2500 \div 1.20^{5}=1004.69$ |  | M | 1.3a | M1 $P=2500 \div 1.20^{5}$ |
|  |  |  | A | 1.3a | A1 1005 |



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